Total No. of Printed Pages:4

SUBJECT CODE NO: E-13
FACULTY OF ENGINEERING AND TECHNOLOGY
S.E.(All Branches) Examination Nov/Dec 2017
Engineering Mathematics -IV
(OLD)

[Time: Three Hours] **[Max.Marks:80]**

Please check whether you have got the right question paper.

N.B
1) Q. no 1 and 6 are compulsory.
2) solve any two question from remaining of each section
3) Figures to right indicate full marks.
4) Assume suitable data if necessary.

Section A

Q.1 Solve any five from the following 10

a) What are the sufficient conditions for f (z) to be analytic?

b) Find the image of $|z| = 1$ *under the mapping* $w = \frac{1}{z}$

c) Expand $f\ (z) = \sin Z$ *about* $z = \frac{\pi}{4}$ by using Taylor's series

d) Evaluate $\int_0^i ze^{z^2}\,dz$

e) Solve : $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u, u(0, y) = 3e^{-3y}$

OR

Find Z – transform of $F\ (k) = k, k \geq 0$

f) Solve : $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

OR

Find the Z – transform of $c^k \sin\ \alpha k, k \geq 0$

g) State cauchy's residue theorem

h) Determine the poles and the residue at each pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$

Q.2 a) Determine the function $f(z) = ze^{-z}$ is analytic or not 05

b) Evaluate $\oint_c log\ z\ dz$ where C is the standard unit circle 05

c) Evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$ by using residue theorem 05

Q.3 a) If $f(\alpha) = \int_c \frac{3z^2+7z+1}{z-\alpha}\ dz$ where C is the circle 05

$x^2 + y^2 = 4\ find\ the\ value\ of\ \ f(3), f'(1-i) and\ f''\ (1-i)$

b) Show that $u(r,\theta) = e^{-\theta}\cos(\log r)$ is harmonic, find its harmonic conjugate function 05

c) Solve the partial differential equation $\frac{\partial u}{\partial t} = \frac{1}{h^2}\frac{\partial^2 u}{\partial x^2}$,with subject to the condition 05

$$u(0,t) = 0\,, u\,(l\,,t\,) = 0\,, \quad u\,(x,0\,) = \sin\frac{\pi p}{a}x$$

OR

Find Z -= transform of F(k) = $F(k) = 3^k\cos\left(\frac{k\pi}{2} + \frac{\pi}{4}\right),\ \ k\ \geq 0$

Q.4 a) Find and plot the image of rectangular region bounded by x = 0 ; y = 0 ; x = 2 ; y = 1 under the transformation w = iz 05

b) Find the Laurent series expansion of the function $\frac{1}{(z-1)(z-2)}\ in\ the\ region\ 1 < |z-1| < 2$ 05

c) Solve the equation $u_{xx} + u_{yy} = 0$ subject to the conditions 05

$u\,(0,y) = u\,(\pi,y\,) = 0\ \ for\ all\ y\ and\ u(\,x,0\,) = k,\ \ 0 < x < \pi\ and\ u = 0\ when\ y \to \infty$

OR

solve $y(k+2) - 3y\,(\,k+1) + 2y\,(k) = 4^k,\ \ y\,(0) = 0, y(1) = 1$

Q.5 a) Find the bilinear transformation which maps the point $-1, 0\,, 1, in\ z -$ plane onto the points $-1, -i\,, i$ in w – plane 05

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ by calculus of residue 05

c) The vibration of an elastic string is governed by the partial differential equation 05

$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$. The π and the ends are fixed. The initial velocity is zero and initial deflection

$u(x, 0) = 2\,(\sin x + \sin 3x)$ .find the deflection u(x,t) of the vibrating sting for t > 0

OR

Find the inverse z – transform of $\frac{1}{(z-3)(z-2)}$ in the region $2 < |z| < 3$

Section B

Q.6 Solve any five from the following 10

1) State second shifting theorem of Laplace transforms
2) Find Laplace transform of $\sqrt{1 - \sin t}$
3) Find Laplace transform of $(\, e^{-4t} + \log t\,)\, \delta\,(t - 2)$
4) Find inverse Laplace transform of $\frac{1}{(s+3)^3}$
5) Find inverse Laplace transform of $\frac{e^{-2s}}{s^2+8s+25}$
6) State inverse convolution theorem of Laplace transform
7) Find Fourier transform of $f(x) = e^{-ax}, x > 0$
   $= e^{ax}, x < 0$
8) find Fourier cosine transform of $e^{-\beta x}$

Q.7

a) Find Laplace transform of $\int_0^t \int_0^t \int_0^t t \sin t \;dt\;\;dt\;\;dt$ 05

b) Find inverse Laplace transform of $\frac{1}{s^3+1}$ 05

c) Solve the integral equation $\int_0^\infty f(x) \sin \lambda x \, dx = e^{-\lambda}, \;\; \lambda > 0$ 05

Q.8

a) Evaluate $\int_0^{\infty} e^{-t} \frac{\sin\sqrt{3}\,t}{t}\, dt$ 05

b) Find inverse Laplace transform of $2\tanh^{-1} s$ 05

c) Find Fourier sine and cosine transform $f(x) = 3e^{-2x} - 7e^{-3x}$ 05

Q.9

a) Express the function in items of Heaviside unit step function hence find their Laplace transform of 05

$$f(t) = t - 2, 1 < t < 2$$
$$= 4 - t, 2 < t < 3$$
$$= 0 \quad t > 3$$

b) Solve $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = t^2 e^{3t}$, $y(0) = 2, \frac{dy}{dt} = 6\ at\ t = 0$ 05

c) Using Fourier transform , solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial t^2}\ for\ x \geq 0, t \geq 0$ under the given condition 05

$$u = u_0\ at\ t = 0, t > 0\ and\ u(x, 0) = 0, x \geq 0$$

Q.10

a) Find Laplace transform of 05

$$f(t) = 1, \quad 0 < t < 1$$
$$= 0, \quad 1 < t < 2 \qquad if\ f(t) = f(t+3)$$
$$= -1 \quad t > 2$$

b) Solve
$\frac{dx}{dt} = 2x - 3y; \frac{dy}{dt} = y - 2x, where\ x(0) = 8, y(0) = 3$ by Laplace transform method 05

c) Find f(x) if it's Fourier sine transform is $\frac{e^{-a\lambda}}{\lambda}$. 05