

“Please check whether you have got the right question paper.”

- Question number one & six are compulsory.
- Attempt any two questions from the remaining 4 questions from each section.
- Figures to the right indicate full marks.
- Assume suitable data if necessary.

SECTION-A

Q.1

Attempt any five of the following

10

- Define Gamma function & Evaluate $\int_0^{\infty} e^{-x} x^2 dx$.
- Find the value of $\int_0^{\infty} \frac{x^3}{(1+x)^4} dx$.
- Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x dx$.
- Find the mean value of $y = 3x^2$ from $x = 0$ to 2
- The surface area of solid formed the revolution of the curve $y = f(x)$ about x -axis from $x = a$ to $x = b$.
- Change the order of integration $\int_0^1 \int_0^{2\sqrt{x}} f(x, y) dx dy$.
- Evaluate $\int_{x=0}^x \int_{y=0}^{y=x^2} dx dy$
- Evaluate $\int_0^a \int_0^b \int_0^c dx dy dz$.

Q.2

- Evaluate $\int_0^{2a} x\sqrt{2ax - x^2} dx$. 05
- Evaluate $\int_0^4 \int_0^{x^2} e^{\frac{y}{x}} dx dy$. 05
- Find the volume bounded by the cylinder $y^2 = x, x^2 = y$ & the plane $x + y + z = 2$ & $z = 0$. 05

Q.3

- Evaluate $\int_0^{\infty} \sqrt[3]{x^2 e^{-3\sqrt{x}}} dx$. 05
- Change the order of integration $\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} f(x, y) dx dy$. 05
- Find by the double integration the area included between the curves $xy = 1$ & $2x + 2y = 5$. 05

Q.4

- Evaluate $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}} dx$. 05
- Changing into polar coordinate Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy$. 05
- Find the surface of the solid generated by the revolution of the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$ About x -axis. 05

Q.5

- Prove that $\beta(m, n) = \int_0^{\infty} \frac{t^{m-1}}{(1+t)^{m+n}} dt$. 05
- Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-y-x} x^2 dx dy dz$. 05
- Find the RMS value of $3 \sin 2x$ over a period . 05

SECTION-B

Q.6 Attempt any five of the following 10

- a) Define periodic function with example .
- b) If $f(x) = \pi^2 - x^2$ over $(-\pi, \pi)$ then write the value of Fourier coefficients a_0 .
- c) What is the half range cosine series for $f(x)$ in the interval $(0, L)$ & Write its Fourier coefficients.
- d) If $f(x) = x ; x \in (0, 2\pi)$ with period 2π find Fourier coefficients a_n .
- e) Find the rank of AB if $A = \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$.
- f) State the condition for consistency of system of linear homogeneous equation .
- g) Find the characteristic equation & Eigen value of $A = \begin{bmatrix} 9 & -7 \\ 3 & -1 \end{bmatrix}$.
- h) State Cayley –Hamilton theorem .

Q.7 a) Find the Fourier series expansion for $f(x) = \frac{\pi-x}{2}$ in the interval $0 < x < 2\pi$ with period 2π . 05

b) Find the rank of matrix $A = \begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$. 05

c) Check the consistency and solve if possible . 05

$$\begin{aligned} 2x - y - 3z &= 8 \\ -x + 2y + z &= 4 \\ 3x + y - 4z &= 0. \end{aligned}$$

Q.8 a) Find the Eigen value & Eigen vector for smallest Eigen value for matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$. 05

b) Find the half range cosine series for $f(x) = x^2$ in the interval $(0, \pi)$. 05

c) Express $f(x) = \frac{x\pi^2}{2}$ as Fourier series defined in $-\pi < x < \pi$. 05

Q.9 a) Verify Cayley –Hamilton theorem for matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$. 05

b) Find the Fourier series for $f(x) = \begin{cases} l-x & 0 < x < l \\ 0 & l < x < 2l \end{cases}$. 05

c) Check the consistency and solve , 05

$$\begin{aligned} x + y + z &= 0 \\ 2x - y - 3z &= 0 \\ 3x - 5y + 4z &= 0 \end{aligned}$$

$x + 17y + 4z = 0$.

Q.10 a) Find the Fourier series for $f(x) = a - x^2$ in $(-a, a)$. 05

b) Find the half range series for $f(x) = \pi x - x^2$ in the interval $(0, \pi)$. 05

c) Prove that given matrix A is orthogonal if $A = \begin{bmatrix} \frac{-8}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{4}{9} & \frac{-8}{9} \\ \frac{4}{9} & \frac{7}{9} & \frac{4}{9} \end{bmatrix}$. 05