

[Time: Three Hours]

[Max. Marks: 80]

“Please check whether you have got the right question paper.”

- N.B
- Question number one and six are compulsory.
  - Attempt any two questions from the remaining questions in each section.
  - Figures to the right indicate full marks.
  - Assume suitable data, if necessary.

## Section – A

- Q1. Solve any five from the following. 10
- Evaluate  $\int_0^{\frac{\pi}{4}} \cos^3 2t \cdot \sin^2 4t \, dt$ .
  - Evaluate  $\int_0^1 \sqrt{1 - x^2} \, dx$ .
  - Find the RMS value of  $F(x) = e^x + 1$  over the range  $x = 0$  to  $x = 2$ .
  - Evaluate  $\int_0^1 \int_0^{\frac{1}{y}} y e^{xy} \, dx \, dy$ .
  - Change the order of integration  $\int_0^1 \int_1^{e^x} f(x, y) \, dy \, dx$
  - Evaluate  $\int_1^2 \int_0^{\log r} d\theta \, dr$ .
  - Find the volume of solid of revolution generated by revolving the curve whose parametric equation are  $x = 2t + 3$ ,  $y = 4t^2 - g$ , about x-axis, between  $t = \frac{-3}{2}$  to  $t = \frac{3}{2}$ .
  - The surface area of the solid generated by the revolution of the area bounded by the curve  $y = f(x)$ , the x-axis and the ordinate  $x = a$  and  $x = b$ , about the x-axis is -----
- Q.2. a) Evaluate  $\int_0^{\infty} \frac{1}{3^{4x^2}} \, dx$  05
- Evaluate  $\int_0^{\infty} \int_0^{\infty} e^{-x^2(1+y^2)} x \, dx \, dy$  05
  - Find the area of the surface of revolution generated by revolving the curve  $x = y^3$  from  $y = 0$  to  $y = 2$  about y-axis. 05
- Q.3 a) Evaluate  $\int_0^1 \frac{1}{\sqrt[3]{1-x^3}} \, dx$  05
- Evaluate  $\iint_R (5 - 2x - y) \, dx \, dy$ , where R is  $y = 0$ ,  $x + 2y = 3$  and  $x = y^2$ . 05
  - Find the area bounded by  $y = x^2 - 3x$  and the line  $y = 2x$ . 05

Q.4 a) Evaluate  $\int_0^{\infty} \frac{\sqrt{x}}{4+4x+x^2} dx$  05

b) 05

Change the order of integration and evaluate  $\int_0^a \int_y^{\sqrt{ay}} \frac{x}{x^2+y^2} dx dy$ . 05

c) 05

Find the volume bounded by the cylinders  $x^2 + y^2 = 2ax$  and  $z^2 = 2ax$ .

Q.5 a) Evaluate  $\int_1^3 \int_{\frac{1}{x}}^1 \int_0^{\sqrt{xy}} xyz dx dy dz$ . 05

b) 05

Change to polar co-ordinate and evaluate  $\int_0^1 \int_{x^2}^x \frac{dx dy}{\sqrt{x^2+y^2}}$ . 05

c) 05

Find the mean value of  $e^{-x^2-y^2}$  over  $x^2 + y^2 = 1$ .

Section – B

Q.6 Solve any five from the following : 10

a) Find the value of Fourier coefficient  $b_n$ , if  $f(x) = 2x - x^2$  in the interval  $(0,3)$ .

b) If  $f(x) = |x|$  in the interval  $(-\pi, \pi)$  then find  $a_n$ .

c) If  $f(x) = \sqrt{1 - \cos x}$ , in the interval  $0 \leq x \leq 2\pi$ , then find  $a_0$ .

d) Define the Fourier series and Fourier coefficients of  $f(x)$  with period  $2\pi$  in the interval  $(c, c + 2\pi)$ .

e) Verify Cayley – Hamilton for  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

f) If the characteristic equation for the matrix A is  $\lambda^3 - 18\lambda^2 + 104\lambda - 192 = 0$ , then find Eigen values of the matrix A.

g) Find the rank of the matrix  $A = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$ .

h) Examine whether the following vectors are linearly independent or dependent.

$X_1 = (1, 2, 3)$   $X_2 = (2, -2, 6)$ .

Q.7 a) Obtain the Fourier series for the function  $f(x) = 0, -\pi \leq x \leq 0$  05

$= \sin x, 0 \leq x \leq \pi$ .

b) Find Half –range sine series for  $f(x) = a(1 - \frac{x}{l})$  for  $0 \leq x \leq l$ . 05

c) Find the rank of the matrix  $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$  05

Q.8) a) Find the Fourier series for 05

$$f(x) = \pi x, \quad 0 \leq x \leq 1$$

$$= \pi (2-x), \quad 1 \leq x \leq 2$$

b) Find the Fourier series for

$$f(x) = \cos x, \quad -\pi < x < 0$$

$$= -\cos x, \quad 0 < x < \pi$$

c) Test for consistency and solve the system :

$$5x + 3y + 7z = 4, \quad 3x + 26y + 2z = 9, \quad 7x + 2y + 10z = 5.$$

Q.9 a) Find the Fourier series of  $f(x) = e^x, -1 \leq x \leq 0$

$$= e^{-x}, \quad 0 \leq x \leq 1.$$

c) Determine the Eigen values and Eigen vector for the highest Eigen value of the matrix  $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

d) Determine the value of  $\lambda$  so that the. Equation  $2x + y + 2z = 0, \quad x + y + 3z = 0, \quad 4x + 3y + \lambda z = 0$  have non – zero solution.

Q.10a) Obtain a Half – range cosine series for  $f(x) = 2x - 1, \text{ for } 0 < x < 1.$

b) Verify Cayley – Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$  and use it to find  $A^{-1}$

c) Examine whether the following vectors are linearly independent or dependent

$$X_1 = [1, 2, 3]^T, \quad X_2 = [3, -2, 1]^T, \quad X_3 = [1, -6, -5]^T$$