DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD - 402 103
Semester Examination: December – 2017

Branch: All Courses **Semester: I**

Subject with Subject Code: Engineering Mathematics-I (MATH101) **Marks: 60**

Date: 11/12/2017 **Time: 3 Hrs.**

Instructions to the Students:-

1. Each question carries 12 marks.
2. Attempt **any five** questions of the following.
3. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

(Marks)

Q.1. (a) For what value of λ the following system of linear equations is consistent and solve it completely in each case: **(06)**

$$x+y+z=1,\; x+2y+4z=\lambda,\; x+4y+10z=\lambda^2 .$$

(b) Find the eigen values and the corresponding eigen vectors for the matrix

$$A=\begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$ **(06)**

Q.2. (a) If $y=\sin px+\cos px$, then prove that $y_n=p^n\left[1+(-1)^n\sin(2px)\right]^{\frac{1}{2}}$. **(04)**

(b) If $y=e^{a\cos^{-1}x}$, then prove that **(04)**

$$(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+a^2)y_n=0 .$$

(c) Expand y = $\log(\cos x)$ about the point $x=\frac{\pi}{3}$ up to third degree by using Taylor's series. **(04)**

Q.3. Attempt **Any Three:** **(12)**

(a) If $x^x y^y z^z = c$, then prove that at point $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \ln ex)^{-1}$.

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ **,** prove that $x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = \sin 4u - \sin 2u$.

(c) If $x^2 = au + bv, y^2 = au - bv$ and $z = f(u, v)$, then prove that
$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2\left(u \frac{\partial z}{\partial u} + v \frac{\partial z}{\partial v}\right)$.

(d) If $u = \sin\left(\frac{x}{y}\right)$ where $x = e^t, y = t^2$, then find $\frac{du}{dt}$.

Q.4. Attempt **Any Three:** **(12)**

(a) If $ux = yz, vy = zx, wz = xy$, then prove that J J* = 1 where $J = \frac{\partial(u, v, w)}{\partial(x, y, z)}$ and J* = $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(b) If the sides and angles of a plane triangle vary in such a that its circum-radius remains constant, then prove that $\frac{da}{\cos A}$ + $\frac{db}{\cos B} + \frac{dc}{\cos C}$ = 0.

(c) A rectangular box open at the top is to have volume of 32cubic units. Find the dimensions of the box requiring the least material for its construction by Lagrange’s method of undetermined multipliers.

(d) Expand $f(x, y) = x^y$ as for as second degree in the powers of $(x - 1)$ and $(y - 1)$ using Taylor's theorem.

Q.5. Attempt **Any Three:** **(12)**

(a) Change the order of integration and evaluate I = $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y} dxdy$.

(b) Use elliptical polar form to evaluate I = $\iint_R xy \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^{\frac{n}{2}} dxdy$, where R is the region of ellipse in positive quadrant.

(c) Use spherical polar transformation to evaluate I = $\int_0^\infty dx \int_0^\infty dy \int_0^\infty \frac{dz}{(x^2 + y^2 + z^2)^2}$.

(d) Find the centroid of the positive loop of the curve $r^2 = a^2 \cos 2\theta$.

Q.6. **(a)** Test the convergence of the series $\sum_{1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2} \right)$. **(04)**

(b) Test the convergence of the series $\sum_{1}^{\infty} \frac{(n+1)^n x^n}{n^{(n+1)}}$. **(04)**

(c) Test the absolute convergence of the series $\sum_{2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$. **(04)**
