Total No. of Printed Pages:03

SUBJECT CODE NO:- H-292
FACULTY OF SCIENCE AND TECHNOLOGY
F. E. (All)
Engineering Mathematics - I
(REVISED)

[Time: Three Hours] **[Max. Marks: 80]**

Please check whether you have got the right question paper.

N.B
1) Use of non-programmable calculator is allowed
2) Q. no.1 and Q. no. 6 are compulsory
3) Solve any two question from Q. nos. 2,3,4 and 5
4) Solve any two question from Q. nos. 7,8,9 and 10

SECTION A

Q.1 Attempt the following (Any five): 10

a. State condition for consistency of a system of homogeneous equation.

b. Define Eigen values and Eigen vectors.

c. Find rank of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

d. Define linear transformation.

e. Find modulus and amplitude of $Z = 1 - i$

f. Simplify $\frac{(cos\theta + isin\theta)^4}{(cos\theta - isin\theta)^3}$

g. State De-Moivre's theorem.

h. Find general value of $log(-10)$.

Q.2 a. Find rank of matrix A by reducing it to its normal form. 05

$A = \begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$

b. Find Eigen values and Eigen vector corresponding largest Eigen value of following matrix. 05

$A = \begin{bmatrix} 0 & -1 & -2 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

c. The centre of regular hexagon is at origin and one vertex is $\sqrt{3}+i$ on Argrand's diagram, determine the other vertices 05

Q.3 a. Test for consistency and solve if possible the following system of equations 05
$2x+3y-4z=-2, \quad x-y+3z=4, 3x+2y-z=-5$

b. Verify cayley-Hamilton theorem and find inverse of 05
$A=\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}$

c. Show that all roots of equation $(x+1)^7=(x-1)^7$ are given by $\mp icot\left[k\frac{\pi}{7}\right]$. Where k=1, 2, 3. 05

Q.4 a. Examine for linear dependence or linear independence and find relation if dependence the following set of vectors. 05
$[2,3,-1,-1], \quad [1,-1,-2,4], \quad [3,1,3,-2], \quad [6,3,0,-7]$

b. Separate real and imaginary parts of $\sin^{-1}[e^{i\theta}]$. 05

c. If $cosec\left(\frac{\pi}{4}+ix\right)=u+iv, prove\ that\ (u^2+v^2)^2=2(u^2-v^2)$. 05

Q.5 a. Given the transformation 05
$Y=\begin{bmatrix}1 & -2 & 3\\ 2 & 0 & -3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$ find co-ordinates $[x_1\ \ x_2\ \ x_3] to\ (2,3,0) in\ y.$

b. Separate i^{i^i} into real and imaginary parts. consider only principle values 05

c. If $\tan(\alpha+i\beta)=i, \alpha, \beta$ being real, prove that α is indeterminate and β is infinite 05

SECTION B

Q.6 Attempt the following (Any five): 10

a. Find n^{th} order derivation of $y=\frac{1}{2x+5}$

b. State Maclaurin's theorem and derive series for $tanx$

c. State Cauchy's n^{th} root test

d. Find stationary values of function $x^3y^2(1-x-y)$

e. Find Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}\ if\ u=rcos\theta, v=rsin\theta$

f. If $u=\sin\sqrt{\frac{x-y}{x+y}}$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

g. Evaluate $\lim_{x\to 1}\{\frac{\log sinx}{cosx}\}$

h. Derive series for $\log(x+1)$

Q.7 a. Find the n^{th} derivative of $\frac{x}{(x-1)(x-2)(x-3)}$ 05

b. Find $\frac{dy}{dx}, if\ (cosx)^y=(siny)^x$. 05

c. If $u = \sec^{-1}\frac{(x^3+y^3)}{x+y}$ *prove that* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2cotu$ 05

Q.8 a. $Evaluate\ \lim_{x\to 0}\left(\frac{tanx}{x}\right)x^{\frac{1}{2}}.$ 05

b. $if\ u = x+y+z,\quad u^2v = y+z,\quad u^2w = z,\ find\ \frac{\partial(x,y,z)}{\partial(u,v,w)}.$ 05

c. $if\ u = \log(x^3+y^3+z^3-3xyz)$ show that 05

1. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$
2. $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$

Q.9 a. Prove that $cosx\ \ coshx = 1 - \frac{2^2x^4}{4!} + \frac{2^4x^8}{8!} \dots\ \dots\dots\ \dots.$ 05

b. Expand $x^4 - 3x^3 + 2x^2 - x + 1$ in powers of x-3 05

c. If $x + y = 2e^{\theta}cos\emptyset\ and\ x - y = 2ie^{\theta}sin\emptyset$ show that $\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial\emptyset^2} = 4xy\frac{\partial^2 u}{\partial x\partial y}$ 05

Q.10 a. Prove that $\tanh^{-1}x = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots$ 05

b. Test for convergence or divergence of $\sum\frac{n^2(n+1)^2}{n!}.$ 05

c. Divide 24 into three parts such that the continued product of first, square of second and cube of third may be maximum. 05